

# Recombination Methods for Jets in $p\bar{p}$ Collisions

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## Abstract

A jet algorithm must specify how to (re-)combine different partons or towers into a single four-vector. Various recombination schemes have been used experimentally to examine the transverse energy profile of jets in hadron colliders. Generally, the data is insensitive to which scheme is used. However, we argue that the recombination scheme previously used by the DØ collaboration is expected to have large perturbative corrections and should not be used for the purposes of making a quantitative comparison with fixed-order perturbation theory.

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Hadrons are produced copiously in collisions at  $p\bar{p}$  colliders. Those particles carrying the bulk of an event's energy are usually observed in relatively narrow, collimated sprays, known as *jets*. To make a quantitative comparison between theory and experiment using this observation, one must go beyond a qualitative definition, and give a precise algorithm for defining a jet. We must define the experimental jet algorithm in terms of the measured properties of hadrons in the detector, and simultaneously define a theoretical version at the parton level to be used in perturbative QCD predictions.

Jet algorithms are not unique, and neither of course are the experimental results. Jet definitions in experiments, broadly speaking, are two-step algorithms yielding a set of jet axes for a given event along with an assignment of each particle or calorimeter tower either to a specific jet, or to no jet at all. The partonic analog also yields a set of jet axes and an assignment of partons to specific jets (or again to no jet at all) for each final-state configuration at the given order in perturbation theory. For a jet algorithm to be of any use, however, differential cross sections using it must be reliably predicted in perturbation theory. It must therefore satisfy various criteria, for example infrared-safety, in order to be calculable sensibly order-by-order in perturbation theory.

Both the commonly-used cone algorithms and the hadronic version of the Durham or  $k_T$  algorithm [1] contain a notion of recombination, wherein the four-momenta of two or more particles are combined to yield a single four-momentum. Just as in  $e^+e^-$  collisions, there are various ways to do this. The theoretically most straightforward way is to treat all initial particles or partons as massless, and simply add the four-momenta so that,

$$E_T^{\text{jet}} = \sum_{i \in \text{jet}} E_i / \cosh \eta^{\text{jet}}, \quad (1)$$

and,

$$\begin{aligned} \theta^{\text{jet}} &= \text{atan} \sqrt{\left( \sum_{i \in \text{jet}} E_{xi} \right)^2 + \left( \sum_{i \in \text{jet}} E_{yi} \right)^2} / \sum_{i \in \text{jet}} E_{zi}, \\ \eta^{\text{jet}} &= -\ln \left( \tan \left( \theta^{\text{jet}} / 2 \right) \right), \quad \phi^{\text{jet}} = \text{atan} \left( \sum_{i \in \text{jet}} E_{yi} / \sum_{i \in \text{jet}} E_{xi} \right). \end{aligned} \quad (2)$$

The commonly-used Snowmass algorithm [2] adds transverse energies, and forms a transverse-energy-weighted combination of the rapidities and azimuthal angles so that

$$E_T^{\text{jet}} = \sum_{i \in \text{jet}} E_{Ti}, \quad (3)$$

and,

$$\eta^{\text{jet}} = \frac{1}{E_T^{\text{jet}}} \sum_{i \in \text{jet}} E_{Ti} \eta_i, \quad \phi^{\text{jet}} = \frac{1}{E_T^{\text{jet}}} \sum_{i \in \text{jet}} E_{Ti} \phi_i. \quad (4)$$

The DØ collaboration has used a seemingly-similar mixture of these two approaches, adding the transverse energies as in eqn. (3), but defining the direction by addition of momenta with eqn. (2). While seemingly similar, we shall see that for *theoretical* reasons, the DØ recombination scheme (unlike the other two) leads to a poor connection between parton- and hadron-level predictions.

The other important aspect of jet recombination schemes is the assignment of particles to the jet. Here we should emphasize that whatever the direction finding algorithm used, one must attempt to apply the *same* assignment algorithm both to experimental data and in theoretical calculations if one is to have any hope of making a sensible comparison. Typically in experimental analyses fixed cones of radius  $R$  are drawn about ‘seed towers’ to determine which particles lie within the jet. A new jet axis is then calculated and the cone moved until a stable jet center is found. The net result is that all towers within radius  $R$  of the final jet direction are included in the jet but the maximal separation between two towers in the jet is  $2R$ . Using a fixed cone with one of the partons as a seed tower in a next-to-leading order calculation, where at most two partons can combine, forces both partons to lie within a fixed maximal separation  $R$  of each other. On the other hand, a fixed cone about the final jet axis in a perturbative calculation corresponds to a variable maximal separation between the two partons depending on their relative energies, angles, and rapidities. It thus corresponds to the use of a *weighted* cone containing the two partons. For example, in the Snowmass algorithm, the maximal separation between the two partons would be

$$\frac{E_T^{\text{jet}}}{\max(E_{T1}, E_{T2})} R \quad (5)$$

or  $2R$  for equal- $E_T$  partons. The distinction between the two types of cones reflects the absence of seed towers between the two partons; since this lack of seed towers between the two partons is an artefact of perturbation theory, we would argue that the correct way to compare theory and experiment is to use a weighted cone and combine all particles within  $R$  of the final jet direction. Therefore, throughout this paper, we will employ a weighted-cone algorithm, where two partons are clustered if they lie within  $R$  (chosen to be 1.0) of the jet axis reconstructed according to the one of the three recombination algorithms under consideration. The next-to-leading order program we use [3] was constructed using the techniques described in refs. [4] and the one-loop matrix elements of ref. [5]. While our discussion will be in the context of cone algorithms, the comments apply to recombination schemes in the  $k_T$  algorithm as well.

A recent DØ paper [6] studied the transverse-energy profile of jets in certain rapidity and  $E_T$  slices. The DØ jet finding algorithm is identical to the Snowmass algorithm, however the final jet axis was then reconstructed using the DØ recombination scheme; thus not all hadrons in the jet will be within the nominal jet radius  $R$  of the jet axis. The integrated transverse-energy density of a jet of radius  $R$  is given by

$$\Psi(r) = \frac{\int_0^r \frac{dE_T}{dr}}{\int_0^R \frac{dE_T}{dr}} = \left\langle \sum_{\text{jets}} \frac{E_T(r)}{E_T(r=R)} \right\rangle_{\text{jets}} \quad (6)$$

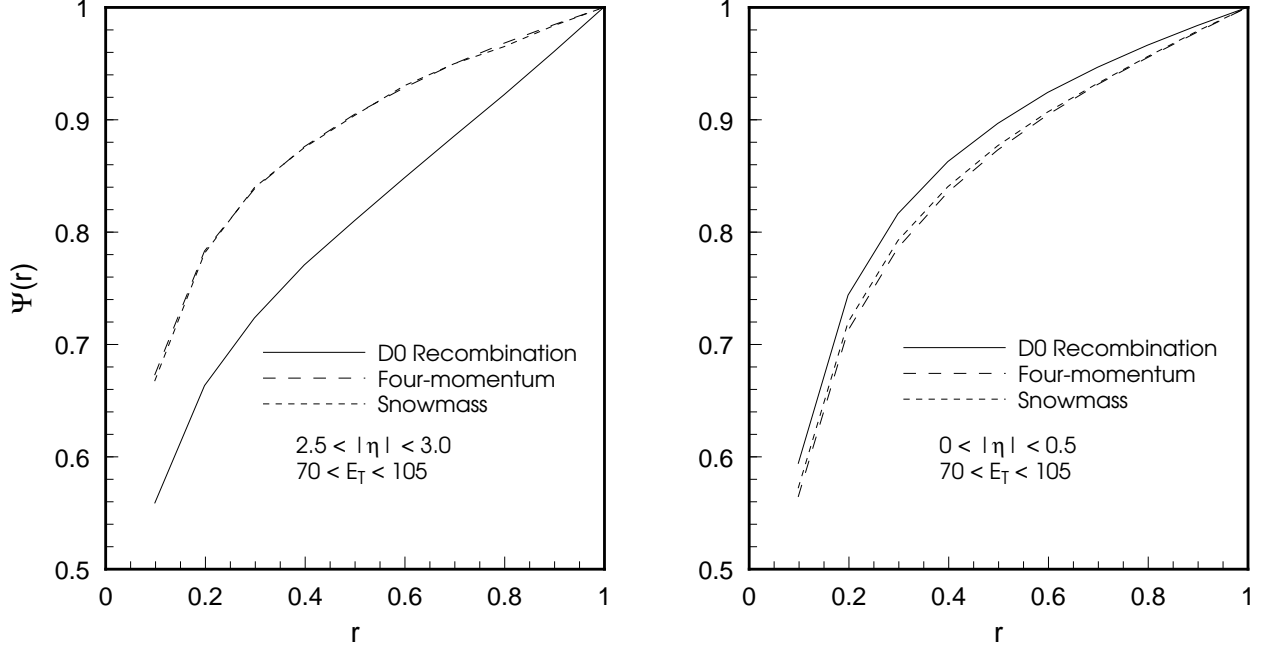


Figure 1. (a) The transverse-energy density profile of jets in the forward region (b) The same quantity for central jets.

where  $E_T(r)$  is the transverse energy within a cone of size  $r = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$  of the jet axis, and where  $\langle \rangle_{\text{jets}}$  denotes averaging over all jets in the given  $E_T \times \eta$  bin within the event sample. This study claimed to find large discrepancies between the data and next-to-leading order calculations.

In many of the comparisons in ref. [6], a fixed-size cone containing the two partons in an NLO calculation was used as the clustering criterion, whereas the experimental data analysis used a Snowmass-type algorithm, which, as we have argued above, should be matched by a weighted-cone theoretical clustering criterion. Indeed, as shown in fig. 5 of ref. [6], an NLO calculation using the Snowmass recombination with a weighted cone agreed reasonably well with the data (at least outside the jet core) for both central and forward rapidities. At smaller values of  $r$  the agreement is less good, but this is understandable since one is then sensitive to large logarithms,  $\ln(r)$ .

The same study also showed that the *experimental* measurements of the jet transverse-energy profile (6) for two different recombination schemes, Snowmass and DØ, are rather similar. However, it was noted in passing that the *theoretical* predictions for the jet transverse-energy profile (6) for these two different recombination schemes are quite different in the forward region (fig. 5 of ref. [6]). The same difference is also shown in figure 1 of the present paper. One may wonder whether this large difference is a result of the general unreliability of perturbation theory, or whether it reflects the difficulties of making a prediction for one of the two schemes. As we shall see, it is the latter: quantities using the DØ scheme are poorly predicted in perturbation theory, and it thus cannot be used for comparing data to predictions from perturbative QCD. (Iterative cone algorithms are known to have various undesirable features from the point of view of fixed-order perturbation

theory; for example, there is typically a prescription for ‘splitting’ or ‘merging’ partially overlapping jets, configurations that cannot be modelled at next-to-leading order. The difficulties with the DØ recombination scheme are additional problems beyond this.)

Any sensible recombination scheme must be infrared-safe: in the limit where two partons become collinear, or where one parton becomes soft, it must yield the same jet axis as would be obtained with one fewer parton. All three recombination schemes satisfy this constraint. However, while it is necessary in order that an observable measured using the recombination scheme be computable reliably in perturbation theory, infrared-safety is not sufficient. To understand the problems with the DØ recombination method, it will be useful to consider the following two quantities, the event’s ‘jet momentum fraction’

$$x_{\text{jet}} = \max_{\sigma=\pm} \sum_{\text{jets } j} E_{Tj} e^{\sigma \eta_j} , \quad (7)$$

and the per-jet fractional energy defect,

$$\begin{aligned} \Delta\varepsilon &= \left( E^{\text{jet}} - \sum_{i \in \text{jet}} E_i \right) / \sum_{i \in \text{jet}} E_i \\ &= \left( E_T^{\text{jet}} \cosh \eta^{\text{jet}} - \sum_{i \in \text{jet}} E_{Ti} \cosh \eta_i \right) / \sum_{i \in \text{jet}} E_{Ti} \cosh \eta_i \end{aligned} \quad (8)$$

Were we to sum over the partons instead of summing over jets in eqn. (7), we would obtain the maximum of the two initial-state parton momentum fractions  $x_{1,2}$ , a quantity that is thus strictly less than one. The quantity defined of course depends on the jet algorithm used to cluster partons into jets, and may therefore exceed one. However, events with  $x_{\text{jet}} > 1$  are guaranteed to have large higher-order corrections, because they are forbidden at lowest order (where  $x_{\text{jet}} = \max(x_1, x_2)$ ).

In figure 2, we show the distribution in  $x_{\text{jet}}$  values. The distributions computed using both the Snowmass and four-momentum recombination schemes die off as  $x_{\text{jet}} \rightarrow 1$ , avoiding at least this source of potentially large higher-order corrections; the DØ recombination scheme, in contrast, does generate events with  $x_{\text{jet}} > 1$ . While the weight of such events may appear to be small in this plot, in certain regions of phase space, they can lead to dramatic effects.

For events with a jet in the forward region,  $x_{\text{jet}}$  is typically quite close to the total reconstructed energy (scaled by  $2/\sqrt{s}$ ), and  $x_{\text{jet}} > 1$  corresponds roughly to events where one jet energy exceeds one-half the available center-of-mass energy. Since the jets can be treated as effectively massless, this is unphysical.

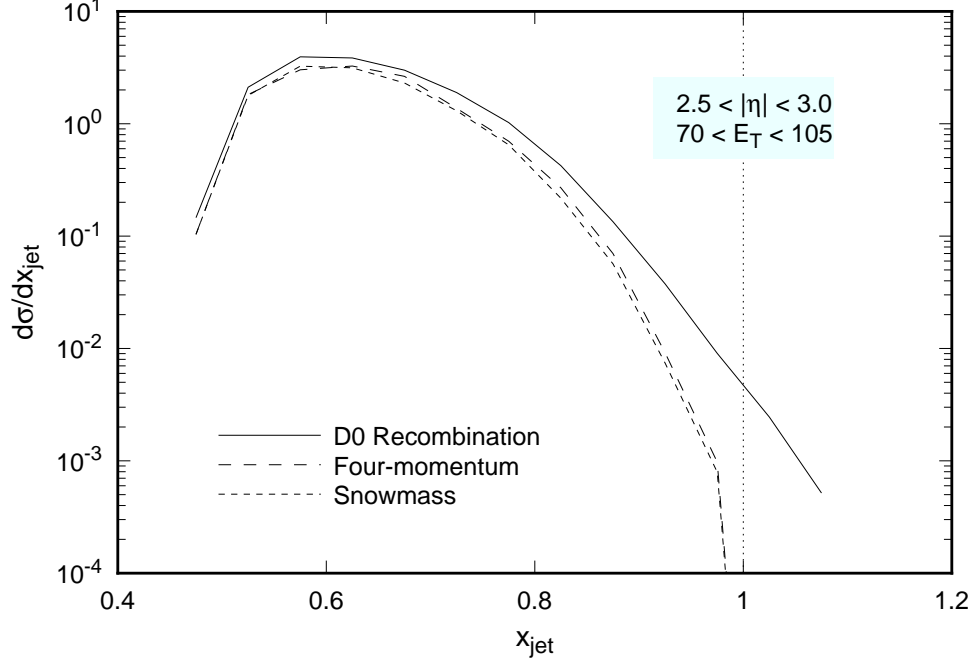


Figure 2. The single-jet inclusive differential distribution in  $x_{\text{jet}}$ , for the forward region. The lowest-order kinematic boundary  $x = 1$  is indicated by a dotted line.

In the DØ scheme, the fractional energy defect is given by the following expression,

$$\Delta\varepsilon = \frac{(E_{T1} + E_{T2})\sqrt{(E_{T1} \cosh \eta_1 + E_{T2} \cosh \eta_2)^2 + 2E_{T1}E_{T2}(\cos \Delta\phi - \cosh \Delta\eta)}}{(E_{T1} \cosh \eta_1 + E_{T2} \cosh \eta_2)\sqrt{(E_{T1} + E_{T2})^2 + 2E_{T1}E_{T2}(\cos \Delta\phi - 1)}} - 1 \quad (9)$$

For small  $\Delta\phi$  and  $\Delta\eta$ , this is

$$\Delta\varepsilon \simeq \frac{E_{T1}E_{T2}}{2(E_{T1} + E_{T2})^2} \left( -\frac{\Delta\eta^2}{\cosh^2 \eta^{\text{jet}}} + \Delta\phi^2 \tanh^2 \eta^{\text{jet}} \right) + \text{higher order} \quad (10)$$

In the forward region,  $\cosh \eta^{\text{jet}} \gg 1$ , while  $\tanh \eta^{\text{jet}} \sim 1$ , so that this quantity is essentially positive; furthermore, it increases rapidly as the two partons in the leading perturbative approximation to the jet move apart in azimuthal angle. (On the other hand, in the central region,  $\tanh \eta^{\text{jet}} \sim 0$ , so that  $\Delta\varepsilon$  is negative.) As an example, we can consider a two partons with equal transverse energies  $E_T$ , and  $\eta_1 = \eta_2 = 2.5$ , and  $\Delta\phi_{12} = \pi/2$  (with a cone radius  $R = 1$ , the maximum azimuthal separation for two such partons within a cluster is 1.778 radians); they will be clustered to form a jet of transverse energy  $2E_T$  at  $\eta = 2.84$ . For such an event, the fractional energy defect will be  $\Delta\varepsilon = 0.40$ : the jet's energy will be overestimated by 40%! In contrast, the same two partons would reconstruct a jet of transverse energy  $2E_T$  but  $\eta = 2.5$  using the Snowmass recombination, or a jet at  $\eta = 2.84$  but with transverse energy  $\sqrt{2}E_T$  using four-momentum

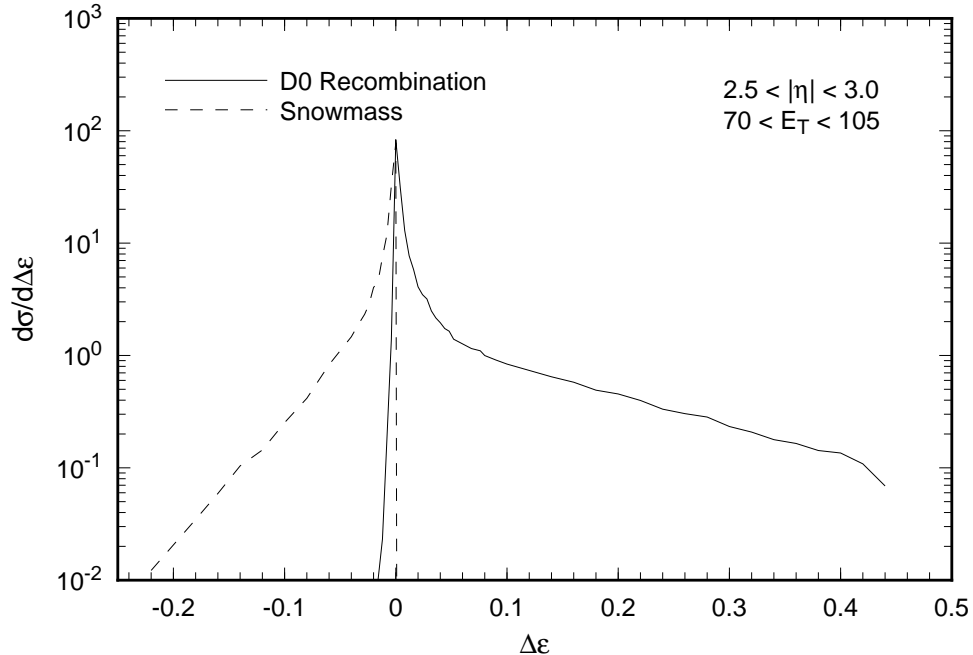


Figure 3. The single-jet inclusive differential distribution in  $\Delta\epsilon$ , for the forward region. The distribution for the four-momentum recombination scheme is a delta function at  $\Delta\epsilon = 0$ , and is not shown on the plot.

recombination. The contribution of such configurations can readily be seen in figure 3; with the DØ recombination scheme, there is a substantial tail of events with  $\Delta\epsilon > 0$ .

So long as  $\Delta\epsilon \sim 0$  on average, that is on average, the reconstructed jet energy is roughly the same as the sum of the parton energies inside it, one might believe that cross sections and distributions should not be too different. However, the parton-level cross section is falling rapidly as a function of energy, and thus shifting the assignment of events upwards in energy can have a dramatic effect. For a given  $E_T \times \eta$  bin (and thus for a given reconstructed jet energy), events with  $\Delta\epsilon > 0$  have smaller net partonic energy and therefore a larger weight, than events with  $\Delta\epsilon \sim 0$ . As the analytic results above show, the energy defect in the DØ recombination scheme is not only positive in the forward region, but increases substantially as the two partons move apart in azimuthal angle. This is reflected in the fact that the reconstructed rapidity grows as the two partons move apart; since the cross section at fixed  $E_T$  falls rapidly as a function of  $\eta$  in the forward region, these events will move to a region of much smaller cross-section, and as a result will carry a disproportionately large weight.

This increase in the energy defect at large separations can also be seen in figure 4(a). In contrast, the four-momentum recombination scheme of course has identically zero defect, while the Snowmass scheme has a negative defect. In further contrast, as shown in figure 4(b), the defects for both Snowmass and DØ recombination are negative in the central region. (One might worry

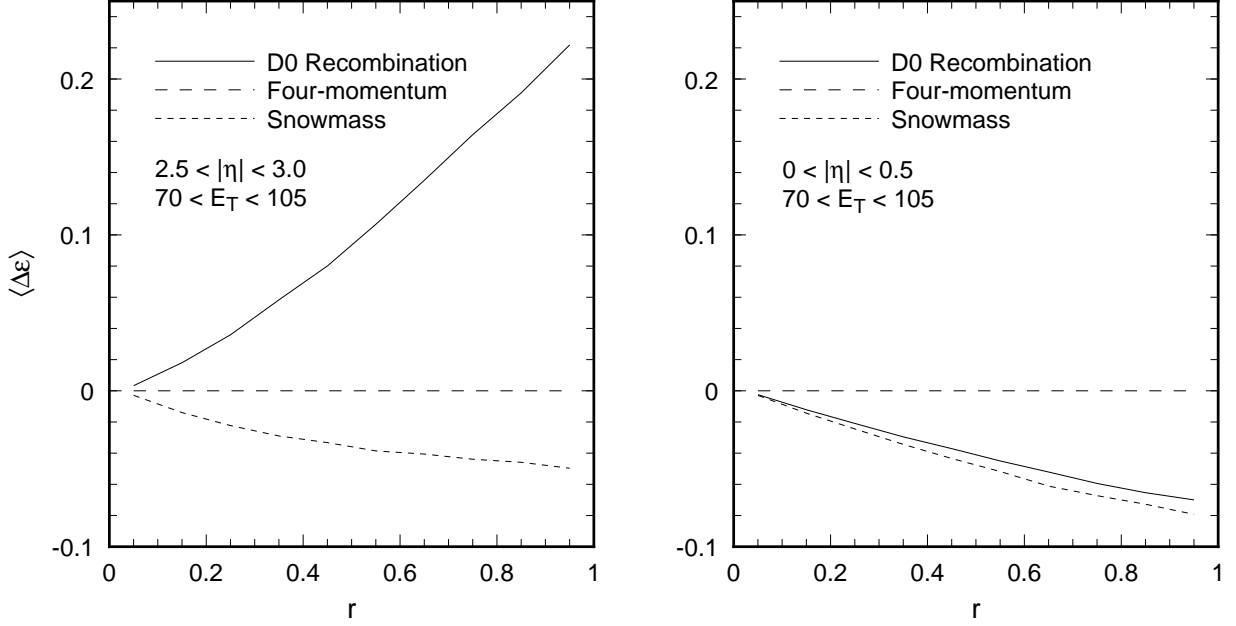


Figure 4. (a) The average energy defect as a function of the distance  $r$  from the jet axis, in the forward region. (b) The same quantity in the central region.

about such negative defects; however, it is physically much more reasonable for an algorithm to lose energy (for example, through ‘leakage’) than it is for it to find more energy than was put in. So long as this negative defect is not too large or rapidly varying in a distribution, it should not have much impact on cross sections or distributions.)

The increasing energy defect at increasing separation means that at larger separation, the differential cross-section within the jet, or equivalently the transverse energy density within the jet, will be over-estimated at large  $r$ , since we will get an increasing contamination of events with a large weight. As a result, at the parton level, jets will appear much broader. This is precisely what is seen in figure 1.

More generally, any distribution across which the average  $\Delta\epsilon$  varies substantially will not be accurately computed in perturbation theory, since different parts of the distribution will suffer different degrees of ‘contamination’ from events of anomalously large weight. This difficulty is a reflection of the fact that a next-to-leading perturbative calculation attempts to model the properties of an average jet by a weighted sum over a statistical ensemble of two-parton configurations. (It should be noted, in fact, that such calculations are only next-to-leading when applied to distributions of the jets in an event; since a leading-order jet calculation has no internal structure, NLO calculations produce the leading non-trivial calculation of jet structure.). In particular, the jet transverse-energy profile  $\Psi(r)$  emerges as the average over a set of very unsmooth distributions,

$$\Psi(r) = \left\langle \sum_{i \in \text{jet}} \frac{E_{Ti}}{E_{T\text{jet}}} \delta(r - r_i) \right\rangle_{\text{jets}} \quad (11)$$



This will succeed only so long as the different configurations of two-parton ensembles are treated uniformly by the jet algorithm. The DØ recombination scheme fails this criterion, since two-parton configurations with large  $\Delta r$  are more likely to contain contributions with  $\Delta\varepsilon$  substantially larger than zero.

As one goes to higher orders in perturbation theory, one will find jets with an increasing number of partons. Although there will be large jet-to-jet fluctuations about the average, the transverse-energy profile of each *individual* jet will still become less lumpy than at the leading non-trivial order, and in particular will contain contributions at many different values of  $r$ . This will tend to smear out the  $\Delta\varepsilon$  distribution shown in figure 4. On average, that is,

$$\langle\Delta\varepsilon\rangle(r)=\left\langle\Delta\varepsilon_{\text{jet}}\sum_{i\in\text{jet}}\frac{E_{Ti}}{E_{T\text{jet}}}\delta(r-r_i)\right\rangle_{\text{jets}}\quad(12)$$

(shown in figure 4), will increasingly receive contributions across the whole range of  $r$  from each event as the number of partons in the jet increases; and this will lessen the variations in  $\Delta\varepsilon$  as one moves from smaller to larger  $r$ . Equivalently, one expects the jet-to-jet fluctuations in  $\Delta\varepsilon$  to become smaller, and as this happens, the shape will be less distorted by the effects considered in this paper; but this of course means that it may be substantially different from the lowest non-trivial order prediction given here. One could quantify these differences by comparing the sort of NLO calculation considered here, with a calculation using a parton shower Monte Carlo such as HERWIG [7]. We expect that the difference between an NLO calculation and HERWIG would be small for either the four-momentum or the Snowmass recombination schemes, but large for the DØ recombination scheme. (Most configurations produced in parton-shower calculations, or in experimental data, will consist of a single jet core centered on the eventual jet axis, surrounding by softer radiation as one moves outward. Such calculations will also produce configurations with two widely-separated jet cores inside the fixed cone. The fate of such configurations depends on the prescriptions for ‘splitting’ and ‘merging’ jets within the jet algorithm. If they are eventually classified as single jets, they will distort jet shapes measured using DØ recombination in a manner similar to that found for two-parton configurations in this paper. However, since such configurations involve an additional wide-angle emission, they will be suppressed by a factor of  $\mathcal{O}(\alpha_s)$ , and the distortion if they are retained will be correspondingly much smaller. This has been studied by Abbott [8] who showed that the HERWIG predictions for the transverse energy profile using the different schemes are almost identical.)

To summarize, although the data [6] seem relatively insensitive to the choice of recombination scheme, the DØ recombination scheme is not as perturbatively stable as either the four-momentum or Snowmass recombination schemes and should be discarded for the purposes of making a quantitative comparison with fixed order perturbation theory.

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